Full-wave Modeling of Wedge Absorbers

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***Abstract*—A transmission-line-based method has been used to model the periodic structures. Using this method, different types of wedge absorbers have been simulated and their simulated reflectivity behaviors have been reported.**

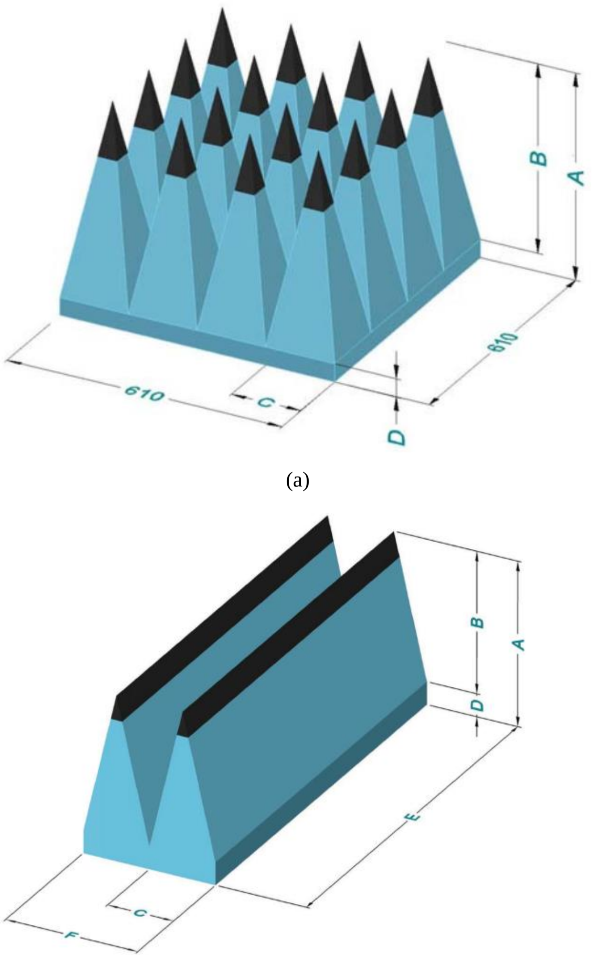
Keywords—2-dimetional periodic structures; Wedge absorbers; Anechoic chambers;

I. INTRODUCTION

There are different applications where the radiation level of an electromagnetic wave is needed to be controlled or reduced. One way to achieve a functional passive control of the radiation level is by the use of electromagnetic-wave absorbers. The absorption efficiency is not only gained by the material characteristics of the base material but also by the geometrical shape of the absorber specifically for wideband absorbers. One of the main applications of wideband absorbers is in anechoic chambers [1]. In anechoic chambers, the walls of the chamber are covered with different absorbing panels each of which have different geometrical shapes. Two major groups of wideband absorbers are the wedge and pyramidal absorbers as shown in Fig. 1. This figure illustrates the general configuration of the problem under investigation as an infinite periodic structure in either one or two directions.

One of the main behavioral characteristics of these absorbers is their reflectivity number. Reflectivity is defined as the power ratio between the reflected wave and the incident wave when the absorber-under-investigation is backed by an infinite PEC wall. Note that, the wave impinging on the lined wall should be a plane wave carrying a specific microwave power per its unit area. In this case, the reflectivity as a measure of the absorber efficiency is a function of incident angle, frequency and in practical cases the angular distribution of the wave impinging on the lined wall. The main focus of the present paper is on the modeling of the wedge absorbers as 1-dimensional periodic structures and investigation of plane-wave reflectivity when an infinite PEC wall is lined with a specific absorber.

There are different ways to model these type of absorbers depending on the application and order of accuracy needed. One way which is very efficient in terms of calculation cost and benefits from low complexity is the average medium technique. In this technique, the pyramidal or wedge absorber is divided into many layers along the vertical direction. A volumetric average value for electric permittivity and magnetic permeability is then assigned to each layer. The resulting problem to be solved is a simple multilayer structure instead of the more complex case of periodic structure. The main benefit of this technique is the possibility of having an analytical closed-form solution which makes the calculation time very short.



(a)

(b)

Fig. 1. Schematic view of the 2 main wideband absorbers used to line the walls of an anechoic chamber; (a) Pyramidal Absorber. (b) Wedge absorber.

However, due to the simple assumption behind the average medium method, the higher order modes of diffraction which are a direct result of the periodic nature of the problem are not taken into account, resulting in low accuracies for some applications.

In order to include the effect of higher-order modes into account, a full-wave method needs to be chosen. The full-wave methods can be divided into two groups:

1- Fully numerical methods

2- Semi-analytical methods

The method used in this paper to model the wedge absorbers is a semi-analytical method which is explained in section II. The simulation results and a discussion on the results are presented in section III. Finally, an outlook on the future research activities will conclude the paper.

II. Modeling method

The method, used in this study for modeling wedge absorbers, is based on the transmission-line formulation for periodic structures. This method starts in a similar way as the average medium technique, meaning that the tapered structure of the absorber is divided into several thin layers. A concatenation of the layers forms the total geometry of the absorber. Fig. 2 shows the geometry of the basic layer analyzed for evaluating each of the pyramidal and wedge absorbers. Note that in this paper only the wedge absorbers are investigated. Hence, the corresponding layer geometry is the one shown in Fig. 2(b).

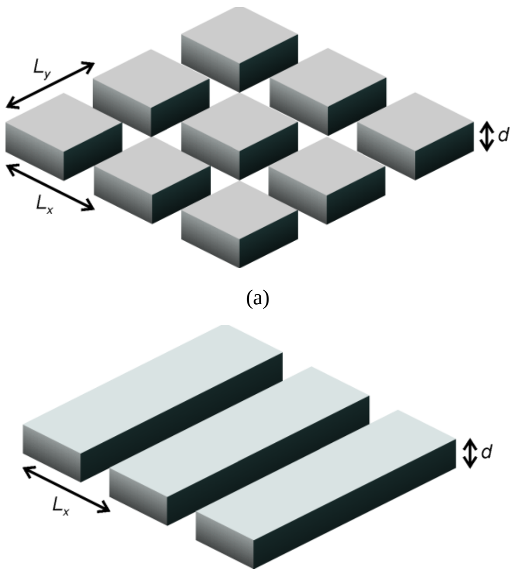


Fig. 2. Schematic illustration of the basic layers forming the two considered absorbers; (a) Pyramidal Absorber. (b) Wedge absorber.

As mentioned before, the average medium technique evaluates each layer by considering a volumetric average permittivity and permeability for each layer. The method we use in this study calculates the properties of each layer using a transmission line formulation for each layer, thereby maintaining the periodic nature of the layers. Subsequently, $\boldsymbol{R}$-matrix algorithm is used to concatenate the layers and acquire the total reflection coefficient based on the reflection coefficient of each layer as defined in Fig. 2.

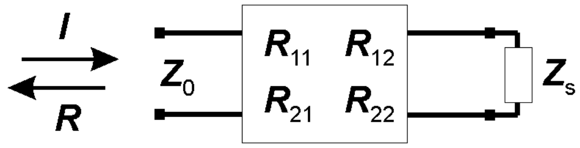


Fig. 3. Dispersion relation for the real and imaginary parts of permittivity.

Having the material properties for each layer, the $\boldsymbol{R}$ matrix for each layer is defined according to:

$$\begin{pmatrix}\boldsymbol{V}(d)\\ \boldsymbol{V}(0)\end{pmatrix} = \boldsymbol{R}\begin{pmatrix}\boldsymbol{I}(d)\\ \boldsymbol{I}(0)\end{pmatrix} = \begin{pmatrix}\boldsymbol{r_{11}} & \boldsymbol{r_{12}}\\ \boldsymbol{r_{21}} & \boldsymbol{r_{22}}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}(d)\\ \boldsymbol{I}(0)\end{pmatrix}, \quad (1)$$

where $\boldsymbol{V} = \left(\boldsymbol{E}_x, \boldsymbol{E}_y\right)^T$ and $\boldsymbol{I} = \left(\boldsymbol{H}_y, -\boldsymbol{H}_x\right)^T$. The vector $\boldsymbol{E}_x$ is defined as a vector containing all the electric field amplitudes of different diffraction orders along x axis, i.e E_{xmn}. Once the $\boldsymbol{R}$ matrix is found for each layer the total reflection matrix of the absorber is calculated through the implementation of the following recursive equation:

$$\begin{aligned}\boldsymbol{R}_{11}^{j} &= \boldsymbol{r}_{11}^{j} + \boldsymbol{r}_{12}^{j}\boldsymbol{Z}\,\boldsymbol{r}_{21}^{j}\\ \boldsymbol{R}_{12}^{j} &= -\boldsymbol{r}_{12}^{j}\boldsymbol{Z}\,\boldsymbol{R}_{12}^{j-1}\\ \boldsymbol{R}_{21}^{j} &= \boldsymbol{R}_{21}^{j-1}\boldsymbol{Z}\,\boldsymbol{r}_{21}^{j}\\ \boldsymbol{R}_{22}^{j} &= \boldsymbol{R}_{22}^{j-1} - \boldsymbol{R}_{21}^{j-1}\boldsymbol{Z}\,\boldsymbol{R}_{12}^{j-1}\end{aligned}, \quad (2)$$

with $\boldsymbol{Z} = \left(\boldsymbol{R}_{11}^{j-1} - \boldsymbol{r}_{22}^{j}\right)^{-1}$. In the above equations, $\boldsymbol{R}^N$, with N the total number of layers, is the overall reflection matrix of the layers. Note that the z axis points from top to bottom and the j index of each layer increases in the same direction. In other words, $\boldsymbol{R}^1$ is the reflection matrix of a layer with no absorbing material.

Once the total $\boldsymbol{R}$ matrix of the layers is found, the whole tapered structure is replaced with an element characterized by the obtained $\boldsymbol{R}$ matrix, as shown in Fig. 3. Therefore, the reflection coefficient of each diffraction order is calculated as:

$$\boldsymbol{Z}_t = \boldsymbol{R}_{21}(\boldsymbol{Z}_s - \boldsymbol{R}_{11})^{-1}\boldsymbol{R}_{12} + \boldsymbol{R}_{22}, \quad (3)$$

$$\boldsymbol{R}_t = (\boldsymbol{Z}_t + \boldsymbol{Z}_0)^{-1}(\boldsymbol{Z}_t - \boldsymbol{Z}_0), \quad (4)$$

where $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_s$ are the impedance matrices of the material above the absorber and the substrate respectively. The impedance matrix of a homogeneous layer is obtained from the following set of equations:

$$\boldsymbol{Z} = \boldsymbol{T}\begin{pmatrix}\boldsymbol{Z^{TE}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{Z^{TM}}\end{pmatrix}\boldsymbol{T^T}, \quad (5)$$

$$\boldsymbol{T} = \begin{pmatrix}\sin\boldsymbol{\theta} & -\cos\boldsymbol{\theta}\\ \cos\boldsymbol{\theta} & \sin\boldsymbol{\theta}\end{pmatrix}, \quad (6)$$

where $\boldsymbol{Z^{TE}}$ and $\boldsymbol{Z^{TM}}$ are diagonal matrices consisting in the intrinsic impedances of TE and TM modes ($Z_{mn}^{TE} = \omega\mu/k_{z_{mn}}$ and $Z_{mn}^{TM} = k_{z_{mn}}/\omega\epsilon$), respectively. $\sin\boldsymbol{\theta}$ and $\cos\boldsymbol{\theta}$ are diagonal matrices of $\sin\theta_{mn}$ and $\cos\theta_{mn}$ with $\theta_{mn} = \tan^{-1}(k_{y_{mn}}/k_{x_{mn}})$. Eventually, the propagation constants in different directions are obtained from:

$$k_{z_{mn}} = \begin{cases} \sqrt{\epsilon\mu k_0^2 - k_{x_{mn}}^2 - k_{y_{mn}}^2} & if\ \epsilon\mu k_0^2 > k_{x_{mn}}^2 + k_{y_{mn}}^2 \\ -j\sqrt{k_{x_{mn}}^2 + k_{y_{mn}}^2 - \epsilon\mu k_0^2} & if\ \epsilon\mu k_0^2 < k_{x_{mn}}^2 + k_{y_{mn}}^2 \end{cases} \quad (7)$$

where $k_{x_{mn}} = k_x + 2\pi m/L_x$ and $k_{y_{mn}} = k_y + 2\pi n/L_y$. The values of k_x and k_y are obtained from the transverse propagation constants of the incident plane wave.

The remaining step is to obtain the $\boldsymbol{R}$ matrix for each layer. This is indeed where the transmission-line (TL) modeling of a periodic structure is beneficial. For a detailed description of the TL formulation of a periodic slab, we refer the reader to [2, 3]. The equivalent voltage and current vectors in the periodic structure is expressed through eigenvectors $\boldsymbol{P}$ and $\boldsymbol{Q}$, and eigenvalues k_z as

$$\begin{pmatrix} \boldsymbol{V}(z) \\ \boldsymbol{I}(z) \end{pmatrix} = \begin{pmatrix} \boldsymbol{PX}^+ & \boldsymbol{PX}^- \\ \boldsymbol{QX}^+ & -\boldsymbol{QX}^- \end{pmatrix} \begin{pmatrix} \boldsymbol{c}^+ \\ \boldsymbol{c}^- \end{pmatrix}, \quad (8)$$

where $\boldsymbol{c}^+$ and $\boldsymbol{c}^-$ are the amplitude vectors of forward and backward waves and $\boldsymbol{X}^{\pm} = \boldsymbol{e}^{\mp j\boldsymbol{k}_z z}$. The detailed formulation for calculating the eigenvectors $\boldsymbol{P}$ and $\boldsymbol{Q}$, and eigenvalues k_z are presented in [2, 3].

Using the equation (8) for the propagating voltage and current the $\boldsymbol{R}$ matrix of a layer can be found as follows:

$$\begin{aligned} \boldsymbol{r}_{11}^j &= j\boldsymbol{P}(\boldsymbol{Q}\tan \boldsymbol{k}_z d)^{-1} \\ \boldsymbol{r}_{12}^j &= -j\boldsymbol{P}(\boldsymbol{Q}\sin \boldsymbol{k}_z d)^{-1} \\ \boldsymbol{r}_{21}^j &= j\boldsymbol{P}(\boldsymbol{Q}\sin \boldsymbol{k}_z d)^{-1} \\ \boldsymbol{r}_{22}^j &= -j\boldsymbol{P}(\boldsymbol{Q}\tan \boldsymbol{k}_z d)^{-1} \end{aligned} \quad . \quad (9)$$

The above listed equations in conjunction with the pertinent TL formulation to obtain the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{k}_z$ offer a complete set of equations to solve any periodic multilayer structure, including the periodic wedge absorber. In the next section, we use the developed software based on the introduced formulation for modeling the wedge absorbers.

III. SIMULATION RELUSTS

Different wedge absorbers have been investigated in this section. The principal geometry of the wedge absorber under study is the one shown in Fig. 1. The values for the dimensional variables shown in Fig. 1 could be found in Table 1 for different types of wedge absorbers.

For the sake of simplicity, the electrical permittivity of the base material has been chosen as follows:

$$\varepsilon_r = 10^{4.3} f^{-0.43} - j10^{4.2} f^{-0.43}$$

wherein, ε_r is the electrical permittivity and f is the frequency. The real and imaginary parts of the permittivity are depicted in Fig. 4. Note that the material is dispersive and the permittivity approximation is only valid for frequencies bellow 10 GHz. Also, the chosen permittivity does not possess the optimum characteristics for the high-efficiency absorbers.

TABLE I. DIMENSIONAL VALUES FOR DIFFERENT WEDGE ABSORBER MODELS.

	Total height A (cm)	Wedge height B (cm)	Wedge width C (cm)	Base height D (cm)
ECCOSORB WG-4	12.7	8.3	6.8	4.4
ECCOSORB WG-8	20.3	15.2	10.2	5.1
ECCOSORB WG-12	30.5	24.1	10.2	6.4
ECCOSORB WG-20	50.8	43.4	20.3	7.4
ECCOSORB WG-26	66.0	55.8	20.3	10.2

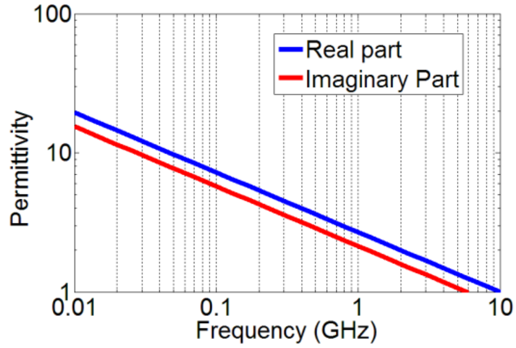
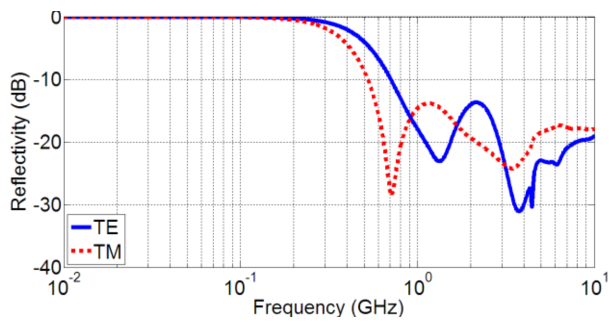


Fig. 4. Dispersion relation for the real and imaginary parts of permittivity.



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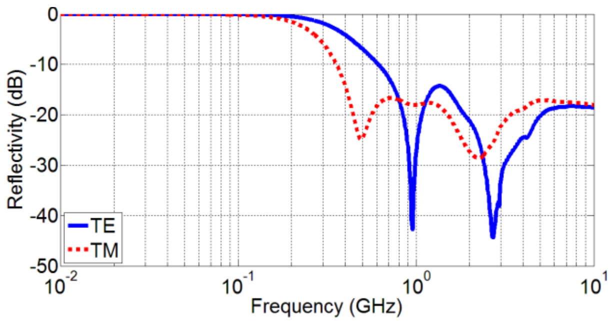

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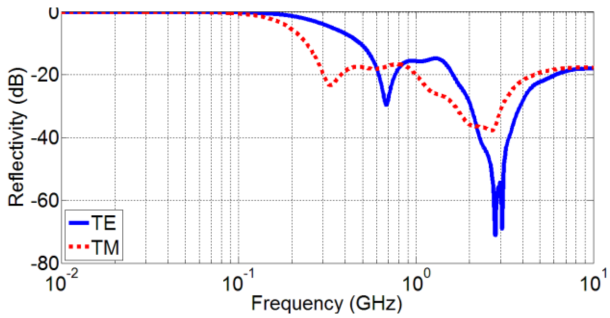

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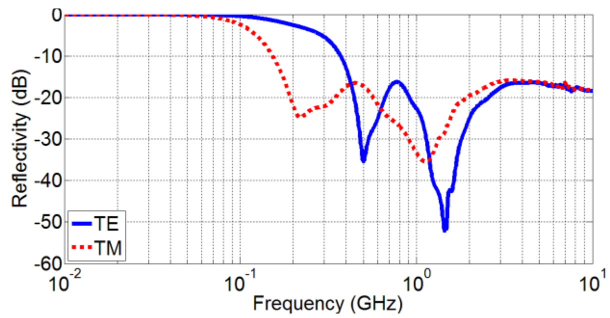

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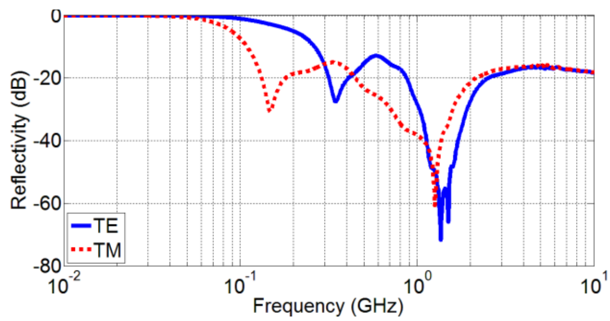
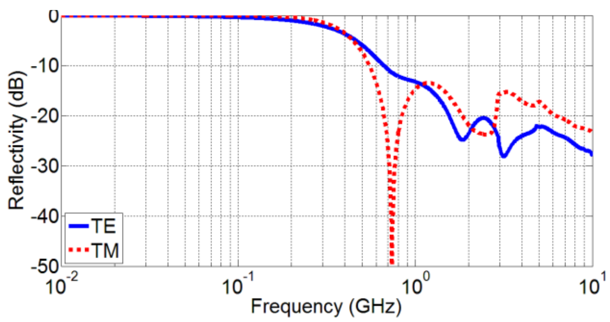


(e)

Fig. 5. Simulated reflectivity for (a) WG-4, (b) WG-8, (c) WG-12, (d) WG-20, and (e) WG-26 for normal incidence and both TE and TM polarizations.

For the normal incidence, the thicker the absorber becomes, the lower the operating frequency goes, see Fig. 5. Note that, the permittivity of the material should be optimized for each of the absorber models, hence, the reflectivity results shown here are not the optimized ones. However, they are good enough to understand how different absorbers behave.



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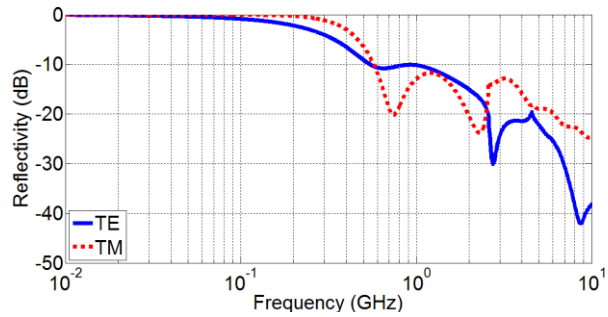

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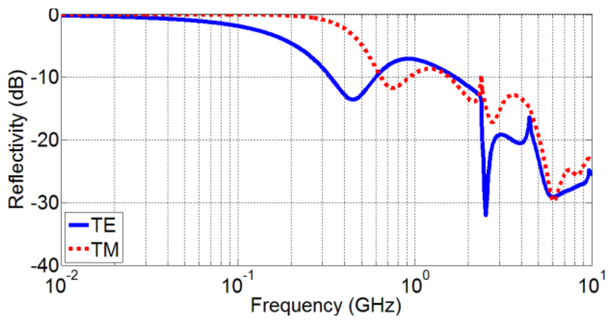

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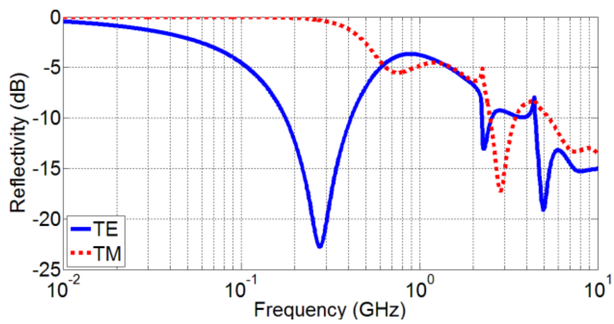

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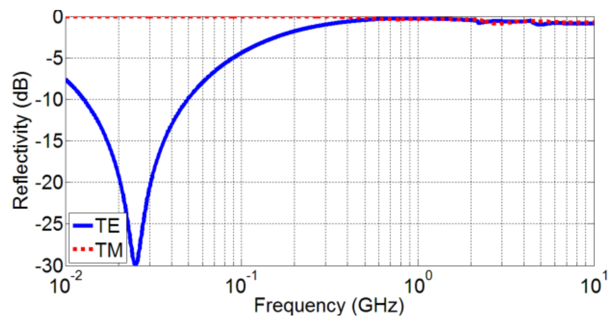

(e)

Fig. 6. Simulated reflectivity of WG-4 for different incident angles; (a) at 30º, (b) 45º , (c) 60º , (d) 75º and (e) 90º.

The next study is to investigate how a wedge absorber behaves when the impinging wave has angles with normal direction. For that, the smallest absorber, i. e. WG-4, was chosen. The simulation results are shown for 5 different incident angles in Fig. 6.

The reflectivity diagram has a deep which moves downwards to the lower frequencies as the incident angle increases. Moreover, the overall reflectivity becomes worse as the incident angle becomes bigger.

IV. CONCLUSION AND FUTURE OUTLOOK

The TL method has been used to model wedge absorbers in a relatively fast and accurate manner. The results are to be compared to the results from other simulation tools/methods and also to the measurement results. Moreover, the TL formulation will be used to model pyramidal absorbers as a 2-dymentional periodic structure.

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