Modeling and Design of Moment Method for Plane Circular Disk


Abstract—This paper is presenting a new and efficient method for computing the charge and current distribution on a circular disk type surface. Integral equation is formulated for the circular loop geometry (taking into the account of Moment Method) to cast the equation for charge and current distribution. The technique involves extension of the conventional Pocklington’s integral equation and the uses of antenna’s effective parameters. Paper consist of a transformation into a single integral equation, and then into a linear system of algebraic equation. Plot of the current distribution along the antenna are finally given and compared with conventional thin wire antenna using matrix inversion technique.

Index Terms—Charge density, Integral equation, Moment method and plane surface.

I. INTRODUCTION

In electrostatics, the problem of finding the potential that is due to a given charge distribution is often considered. In physical situations, however, it is seldom possible to specify a charge distribution. Whereas we may connect a conducting body to a voltage source, and thus specify the potential throughout the body, the distribution of charge is obvious only for a few rotationally symmetric geometries. In this section we will consider an integral equation approach to solve for the electric charge distribution once the electric potential is specified. Some of the material here and in other sections is drawn from [1, 2].

The objective of the Integral Equation (IE) method for radiation or scattering is to cast the solution for the unknown current density, which is induced on the surface of the radiator/scatterer, in the form of an integral equation where the unknown induced current density is part of the integrand. The integral equation is then solved for the unknown induced current density using numerical techniques such as the Moment Method (MM).

The impedance of an antenna can also be found using an integral equation with a numerical technique solution, which is widely referred to as the Integral Equation Method of Moments [3–5]. This method, which in the late 1960s was extended to include electromagnetic problems, is analytically simple, it is versatile, but it requires large amounts of computation. The limitation of this technique is usually the speed and storage capacity of the computer. This method casts the solution for the induced current in the form of an integral (hence its name) where the unknown induced current density is part of the integrand. Numerical techniques, such as the Moment Method [6–8] can then be used to solve the current density.

The current distribution of an antenna depends on many factors like its operating frequency, geometry, method of excitation, and proximity to the surrounding objects. Impedance of an antenna at a point is defined as the ratio of the electric to the magnetic fields at that point; alternatively at a pair of terminals, it is defined as the ratio of the voltage to the current across those terminals also [9].

Most basic approach that can be used to calculate the impedance and current distribution of an antenna is the boundary-value method. The solution to this is obtained by enforcing the boundary conditions (Tangential electric-field components vanish at the conducting surface). Current distribution of an antenna can also be found using an integral equation with a numerical technique solution, which is referred to as the integral equation method of moments which cite the solution for the induced current in the form of an integral equation [10]. This method includes electromagnetic problems which is analytically simple, but requires large computation. In this paper the integral equation method, with a Moment Method numerical solution, will be introduced and used to calculate the current distribution of circular disk. This approach is very general, and it can be used with today’s modern computational methods and equipment to compute the characteristics of complex configurations of circular plate disk type antenna elements. The integral equation is then solved for the unknown induced current density using numerical techniques such as the method of moment (M.O.M).

II. DESIGN PROCEDURE

A circular ring of radius $a$ carries a uniform charge $\rho \cdot C/m$ and is placed on $X-Y$ plane with axis on $Z$ axis.

Using electrostatic for line charge,

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\[ E = \int_{A} \alpha \cdot R \, d\hat{a}_R \]

Where,
\[ d\ell = \alpha d\phi \]
\[ R = \alpha(\hat{a}_\rho) + h\hat{a}_z \]

For Maximum \( E \),
\[ \frac{d|E|}{dh} = 0 \]

This implies that
\[ \left[ h^2 + a^2 \right]^{\frac{3}{2}} \left[ h^2 + a^2 - 3h^2 \right] = 0 \]
\[ a^2 - 2h^2 = 0 \]

Or
\[ h = \pm \frac{a}{\sqrt{2}} \]

Let the charge is uniform distributed on the circular plate, the charge density is,
\[ \rho = \frac{Q}{2\pi a} \]

So that,
\[ E = \frac{Qh}{4\pi \beta \left[ h^2 + a^2 \right]^{\frac{3}{2}}} \hat{a}_z \]

As if \( a \to 0 \)
\[ \frac{Q}{2\pi \beta} \hat{a}_z \]

In general,
\[ E = \frac{Q}{4\pi \beta} \hat{a}_z \]

Which is the same as that of point charge, as expected?

**III. MATHEMATICAL FORMULATION**

The target of the Integral Equation (IE) method for antenna is to cite the solution for the unknown current density, which is induced on the surface of the antenna, in the form of an integral equation where the unknown induced current density is part of the integrand \([12]\). Potential due to a given charge distribution is considered in electrostatics, and now we know that a linear charge distribution creates an electric potential due to ring of radius \( a \).
Similarly for a circular disk of radius \( a \) uniformly charged with \( \rho \) C/m\(^2\). The disk lies on the \( z=0 \) plane with its axis along the \( z \) axis. One can calculate the E field as specified above.

\[
E_{(0,0,h)} = \frac{\rho}{2 \epsilon_0} \left( 1 - \frac{h}{h^2 + a^2} \right) \hat{z}
\]  

(5)

Potential at \((0,0,h)\) will be

\[
V_o = \int_{\text{circular disc}} \frac{\rho}{2 \epsilon_0} \left( 1 - \frac{h(x)}{h(x)^2 + a^2} \right) \, da
\]  

(6)

Where \( x' \) denotes the source coordinates, \( da \) is the path of integration, and \( h(x) \) is the distance from any one ring to the observation point \([13]\).

From calculus we know that,

\[
N = \int_{y} f(y) \, dy = \sum_{k=1}^{N} f(y_k) \Delta y
\]  

(7)

Where the interval \( L \) has been divided into \( N \) units, each having length \( \Delta y \). The circular plate of radius \( a \), is divided into \( N \) uniform ring, each of width \( \Delta \zeta = a/N \). To obtain a solution for these \( N \) amplitude constants, \( N \) linearly independent equations are required \([14]\). These equations may be produced by choosing \( N \) observation points each at the center of each \( \zeta \) length element. Thus equation 6 will become

\[
2\epsilon_0 V_0 \rho \left( 1 - \frac{h(x_1)}{(h(x_1)^2 + a_N)^2} \right) \zeta + \ldots \rho \left( 1 - \frac{h(x_N)}{(h(x_N)^2 + a_N)^2} \right) \zeta
\]  

(8)

The assumption in the above equation is that the unknown charge density \( \rho \) of the \( k^{th} \) segment is constant. Thus in the above equation, we have unknown constant \( \rho, \rho, \ldots, \rho \). Since above equation hold at all points on the surface of the circular plate, we obtain \( N \) similar equation by choosing \( N \) match points at \( h_1, h_2, h_3, \ldots, h_N \) \([15]\). Thus we obtain

\[
2\epsilon_0 V_0 \rho \left( 1 - \frac{h_{m}}{(h_m^2 + a_N)^2} \right) \zeta + \ldots \rho \left( 1 - \frac{h_{N}}{(h_N^2 + a_N)^2} \right) \zeta
\]  

(9)

Equation 7 can put into the matrix form as

\[
[B] = [A][\rho]
\]  

(10)

Where

\[
[B] = \begin{bmatrix}
1 \\
1 \\
. \\
. \\
1
\end{bmatrix}
\]

\[
[A] = \begin{bmatrix}
A_1 & A_2 & A_3 & A_4 & A_5 & \ldots & A_N \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & \ldots & A_{2N} \\
A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & \ldots & A_{3N} \\
. \\
. \\
A_{N1} & A_{N2} & A_{N3} & A_{N4} & A_{N5} & \ldots & A_{NN}
\end{bmatrix}
\]

\[
A_{mn} = \left( 1 - \frac{h_m}{(h_m^2 + a_N)^2} \right) \zeta
\]

\[
[\rho] = \begin{bmatrix}
\rho \\
\rho \\
\rho \\
. \\
\rho \\
\rho
\end{bmatrix}
\]

In equation 10 \( \rho \) is a matrix whose elements are unknown; we can determine using matrix inversion technique \([16]\).

\[
[\rho] = [A]^{-1}[B]
\]

IV. RESULT

During the evaluation of diagonal elements of equation 8 or 9 caution must be exercised. The idea of matching the left hand side of the equation 8 or 9 with the right hand side of the equation at the match points is similar to the concept of taking moments in mechanics. Using equation 8 and letting \( a = \sqrt{m} \) and \( N \geq 10 \), i.e. \( \zeta = a/N \), a MATLAB code can be developed. It inverts the matrix \([A]\) and plots \( \zeta \) against \( h \). The plot is shown in the figure. Program also
Total charge on the wire is \(4.7490 \times 10^{12} \text{C}\).

Total charge on the circular surface is \(9.3500 \times 10^{12} \text{C}\).

V. CONCLUSION

The radiation characteristics of plane circular disk surface have been investigated by applying the method of moment, and current distribution has been obtained. The integral equation for a thin circular plate is derived; some properties of integral equation are presented and are utilized to reduce the computation of integral equation to some sparse matrix notation. The method is computationally attractive, and mathematically formulation is demonstrated through illustrative example. Current distribution of radiating plane circular surface is compared with conventional dipole antenna and demonstrated with a MATLAB based program.

![Figure 3: Plot of current distribution for conventional dipole antenna.](image)

![Figure 4: Plot of current distribution for plane circular surface.](image)

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### References


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### Table I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Numerical Value</th>
</tr>
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<tbody>
<tr>
<td>(\rho)</td>
<td>Charge density</td>
<td>(C/m)</td>
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